

Outcome: Any possible or actual result of the action considered, such as rolling a 5 on a standard number cube or getting tails when flipping a coin.

Event: A desired (or successful) outcome or group of outcomes from an experiment, such as rolling an even number on a standard number cube.

Sample space: All possible outcomes of a situation. For example, the sample space for flipping a coin is heads and tails; rolling a standard number cube has six possible outcomes (1, 2, 3, 4, 5, and 6).

Probability: The likelihood that an event will occur. Probabilities may be written as fractions, decimals, or percents. An event that is guaranteed to happen has a probability of 1, or 100%. An event that has no chance of happening has a probability of 0, or 0%. Events that “might happen” have probabilities between 0 and 1 or between 0% and 100%. In general, the more likely an event is to happen, the greater its probability.

Experimental probability: The probability based on data collected in experiments.

$$\text{Experimental probability} = \frac{\text{number of successful outcomes in the experiment}}{\text{total number of outcomes in the experiment}}$$

Theoretical probability is a calculated probability based on the possible outcomes when they all have the same chance of occurring.

$$\text{Theoretical probability} = \frac{\text{number of successful outcomes (events)}}{\text{total number of possible outcomes}}$$

In the context of probability, “successful” usually means a desired or specified outcome (event), such as rolling a 2 on a number cube (probability of $\frac{1}{6}$). To calculate the probability of rolling a 2, first figure out how many possible outcomes there are. Since there are six faces on the number cube, the number of possible outcomes is 6. Of the six faces, only one of the faces has a 2 on it. Thus, to find the probability of rolling a 2, you would write:

$$P(2) = \frac{\text{number of ways to roll 2}}{\text{number of possible outcomes}} = \frac{1}{6} \text{ or } 0.\overline{16} \text{ or approximately } 16.7\%$$

Example 1

If you roll a fair, 6-sided number cube, what is $P(3)$, that is, the probability that you will roll a 3?

Because the six sides are equally likely to come up, and there is only one 3, $P(3) = \frac{1}{6}$.

Example 2

There are 12 marbles in a bag: 2 clear, 4 green, 5 yellow, and 1 blue. If one marble is chosen randomly from the bag, what is the probability that it will be yellow?

$$P(\text{yellow}) = \frac{5 \text{ (yellow)}}{12 \text{ (outcomes)}} = \frac{5}{12}$$

Example 3

Joe flipped a coin 50 times. When he recorded his tosses, his result was 30 heads and 20 tails. Joe's activity provided data to calculate experimental probability for flipping a coin.

- a. What is the theoretical probability of Joe flipping heads?

The theoretical probability is 50% or $\frac{1}{2}$, because there are only two possibilities (heads and tails), and each is equally likely to occur.

- b. What was the experimental probability of flipping a coin and getting heads based on Joe's activity?

The experimental probability is $\frac{30}{50}$, $\frac{3}{5}$, or 60%. These are the results Joe actually got when he flipped the coin.

Example 4

Decide whether these statements describe theoretical or experimental probabilities.

- a. The chance of rolling a 6 on a fair die is $\frac{1}{6}$.

This statement is theoretical.

- b. I rolled the die 12 times and 5 came up three times.

This statement is experimental.

- c. There are 15 marbles in a bag; 5 blue, 6 yellow, and 4 green. The probability of getting a blue marble is $\frac{1}{3}$.

This statement is theoretical.

- d. When Veronika pulled three marbles out of the bag she got 2 yellow and 1 blue, or $\frac{2}{3}$ yellow, $\frac{1}{3}$ blue.

This statement is experimental.

Problems

1. There are 24 crayons in a box: 5 black, 3 white, 7 red, 2 yellow, 3 blue, and 4 green. What is the probability of randomly choosing a green? Did you respond with an experimental or theoretical probability?
2. A spinner is divided into four equal sections numbered 2, 4, 6, and 8. What is the probability of spinning an 8?
3. A fair number cube marked 1, 2, 3, 4, 5, and 6 is rolled. Tyler tossed the cube 40 times, and noted that 26 times an even number showed. What is the experimental probability that an even number will be rolled? What is the theoretical probability?
4. Sara is at a picnic and reaches into an ice chest, without looking, to grab a can of soda. If there are 14 cans of orange, 12 cans of fruit punch, and 10 cans of cola, what is the probability that she takes a can of fruit punch? Did you respond with an experimental probability or a theoretical one?
5. A baseball batting average is the probability a baseball player hits the ball when batting. If a baseball player has a batting average of 266, it means the player's probability of getting of getting a hit is 0.266. Is a batting average an experimental probability or theoretical?
6. In 2011, 39 people died by being struck by lightning, and 241 people were injured. There were 310,000,000 people in the United States. What is the probability of being one of the people struck by lightning?
7. In a medical study, 107 people were given a new vitamin pill. If a participant got sick, they were removed from the study. Ten of the participants caught a common cold, 2 came down with the flu, 18 got sick to their stomach, and 77 never got sick. What was the probability of getting sick if you participated in this study? Did you respond with an experimental probability or a theoretical one?
8. Insurance companies use probabilities to determine the rate they will charge for an insurance policy. In a study of 300 people that had life insurance policies, an insurance company found that 111 people were over 80 years old when they died, 82 people died when they were between 70 and 80 years old, 52 died between 60 and 70 years old, and 55 died when they were younger than 60 years old. In this study what was the probability of dying younger than 70 years old? Did you respond with an experimental probability or a theoretical one?

Answers

- | | | | |
|--------------------------------|--|---|---|
| 1. $\frac{1}{6}$; theoretical | 2. $\frac{1}{4}$ | 3. $\frac{26}{40}$; $\frac{3}{6}$ | 4. $\frac{1}{3}$; theoretical |
| 5. experimental | 6. $\frac{39+241}{310,000,000}$
$\approx 0.000\ 000\ 903$ | 7. $\frac{10+2+18}{107} \approx 0.28$
experimental | 8. $\frac{55+52}{300} \approx 35.7\%$
experimental |

Measures of central tendency are numbers that locate or approximate the “center” of a set of data—that is, a “typical” value that describes the set of data. Mean and median are the most common measures of central tendency. (Mode will not be covered in this course.)

The mean is the arithmetic average of a data set. Add all the values in a set and divide this sum by the number of values in the set. The median is the middle number in a set of data arranged numerically. An outlier is a number that is much smaller or larger than most of the others in the data set. The range of a data set is the difference between the highest and lowest values of the data set.

For additional information, see the Math Notes boxes in Lessons 1.1.3 and 1.1.4 of the *Core Connections, Course 2* text.

The mean is calculated by finding the sum of the data set and dividing it by the number of elements in the set.

Example 1

Find the mean of this set of data: 34, 31, 37, 44, 38, 34, 42, 34, 43, and 41.

- $34 + 31 + 37 + 44 + 38 + 34 + 42 + 34 + 43 + 41 = 378$
- $\frac{378}{10} = 37.8$

The mean of this set of data is 37.8.

Example 2

Find the mean of this set of data: 92, 82, 80, 92, 78, 75, 95, and 77.

- $92 + 82 + 80 + 92 + 78 + 75 + 95 + 77 + 77 = 748$
- $\frac{748}{9} = 83.1$

The mean of this set of data is 83.1.

Problems

Find the mean of each set of data.

1. 29, 28, 34, 30, 33, 26, and 34.
2. 25, 34, 35, 27, 31, and 30.
3. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.
4. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The median is the middle number in a set of data *arranged in numerical order*. If there is an even number of values, the median is the mean (average) of the two middle numbers.

Example 3

Find the median of this set of data: 34, 31, 37, 44, 38, 34, 43, and 41.

- Arrange the data in order: 31, 34, 34, 34, 37, 38, 41, 43, 44.
- Find the middle value(s): 37 and 38.
- Since there are two middle values, find their mean: $37 + 38 = 75$, $\frac{75}{2} = 37.5$. Therefore, the median of this data set is 37.5.

Example 4

Find the median of this set of data: 92, 82, 80, 92, 78, 75, 95, 77, and 77.

- Arrange the data in order: 75, 77, 77, 78, 80, 82, 92, 92, and 95.
- Find the middle value(s): 80. Therefore, the median of this data set is 80.

Problems

Find median of each set of data.

5. 29, 28, 34, 30, 33, 26, and 34.

6. 25, 34, 27, 25, 31, and 30.

7. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.

8. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The range of a set of data is the difference between the highest value and the lowest value.

Example 5

Find the range of this set of data: 114, 109, 131, 96, 140, and 128.

- The highest value is 140.
- The lowest value is 96.
- $140 - 96 = 44$.
- The range of this set of data is 44.

Example 6

Find the range of this set of data: 37, 44, 36, 29, 78, 15, 57, 54, 63, 27, and 48.

- The highest value is 78.
- The lowest value is 27.
- $78 - 27 = 51$.
- The range of this set of data is 51.

Problems

Find the range of each set of data in problems 5 through 8.

Outliers are numbers in a data set that are either much higher or much lower than the other numbers in the set.

Example 7

Find the outlier of this set of data: 88, 90, 96, 93, 87, 12, 85, and 94.

- The outlier is 12.

Example 8

Find the outlier of this set of data: 67, 54, 49, 76, 64, 59, 60, 72, 123, 44, and 66.

- The outlier is 123.

Problems

Identify the outlier in each set of data.

9. 70, 77, 75, 68, 98, 70, 72, and 71.

10. 14, 22, 17, 61, 20, 16, and 15.

11. 1376, 1645, 1783, 1455, 3754, 1790, 1384, 1643, 1492, and 1776.

12. 62, 65, 93, 51, 55, 14, 79, 85, 55, 72, 78, 83, 91, and 76.

Answers

1. 30.57

2. $30\bar{3}$

3. $86\bar{13}$

4. $106\bar{6}$

5. median 30;
range 8

6. median 28.5;
range 9

7. median 82;
range 47

8. median 107;
range 27

9. 98

10. 61

11. 3754

12. 14

The axis (or axes) of a graph must be marked with equal-sized spaces called intervals. Marking the uniform intervals on the axes is called scaling the axes. The difference between consecutive markings tells the size (scale) of each interval. Note that each axis of a two-dimensional graph may use a different scale.

Sometimes the axis or set of axes is not provided. A student must count the number of usable spaces on the graph paper. How many spaces are usable depends in part on how large the graph will be and how much space will be needed for labeling beside each axis.

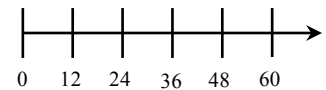
Follow these steps to scale each axis of a graph.

1. Find the difference between the smallest and largest numbers (the range) you need to put on an axis.
2. Count the number of intervals (spaces) you have on your axis.
3. Divide the range by the number of intervals to find the interval size.
4. Label the marks on the axis using the interval size.

Sometimes dividing the range by the number of intervals produces an interval size that makes it difficult to interpret the location of points on the graph. The student may then exercise judgment and round the interval size up (always up, if rounded at all) to a number that is convenient to use. Interval sizes like 1, 2, 5, 10, 20, 25, 50, 100, etc., work well. For more information, see the Math Notes box in Lesson 1.2.2 of *the Core Connections, Course 2* text.

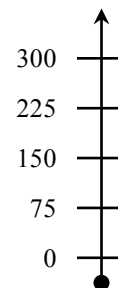
Example 1

1. The difference between 0 and 60 is 60.
2. The number line is divided into 5 equal intervals.
3. 60 divided by 5 is 12.
4. The marks are labeled with multiples of the interval size 12.



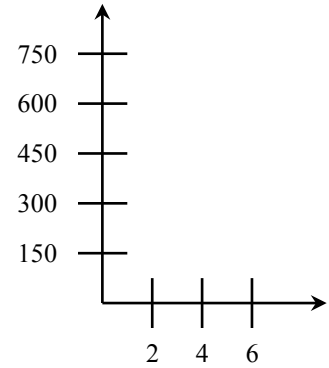
Example 2

1. The difference between 300 and 0 is 300.
2. There are 4 intervals.
3. $300 \div 4 = 75$
4. The axis is labeled with multiples of 75.



Example 3

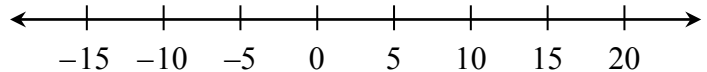
1. The difference on the vertical axis is $750 - 0 = 750$. (The origin is $(0, 0)$.) On the horizontal axis the range is $6 - 0 = 6$.
2. There are 5 spaces vertically and 3 spaces horizontally.
3. The vertical interval size is $750 \div 5 = 150$. The horizontal interval is $6 \div 3 = 2$.
4. The axes are labeled appropriately.



Example 4

Sometimes the axes extend in the negative direction.

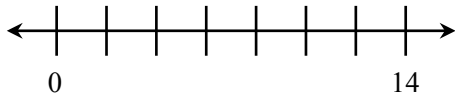
1. The range is $20 - (-15) = 35$.
2. There are 7 intervals along the line.
3. $35 \div 7 = 5$
4. Label the axes with multiples of five.



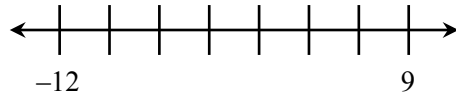
Problems

Scale each axis:

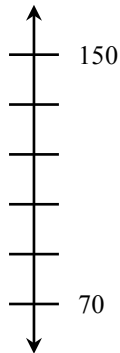
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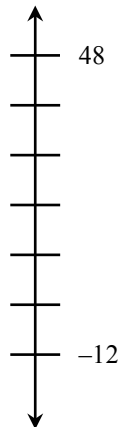
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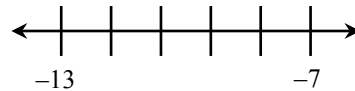
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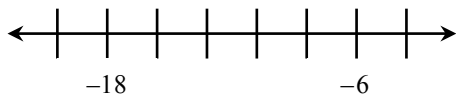
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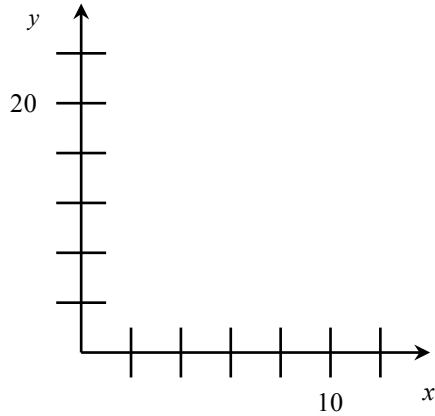
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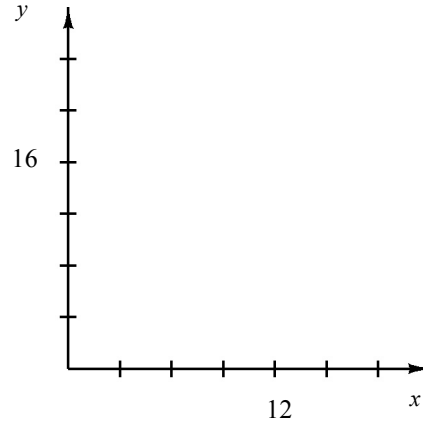
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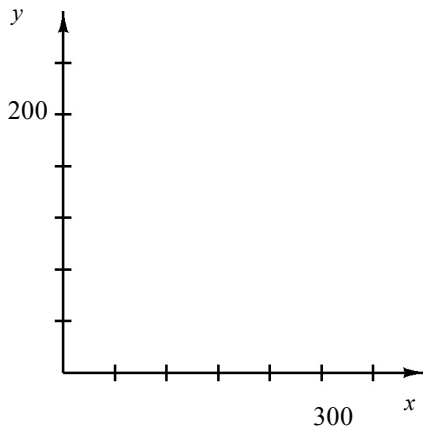
7.



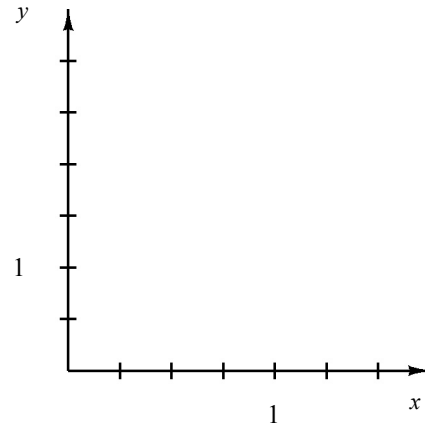
8.



9.



10. Use fractions.



Answers

1. 2, 4, 6, 8, 10, 12

2. -9, -6, -3, 0, 3, 6

3. 86, 102, 118, 134

4. -2, 8, 18, 28, 38

5. -12, -11, -10, -9, -8

6. -20, -16, -14, -12, -10, -8

7. x : 2, 4, 6, 8, 12
 y : 4, 8, 12, 16, 24

8. x : 3, 6, 9, 15, 18
 y : 4, 8, 12, 20, 24

9. x : 60, 120, 180, 240, 360
 y : 40, 80, 120, 160, 240

10. x : $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{1}{2}$
 y : $\frac{1}{2}, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3$

Fractions that name the same value are called equivalent fractions, such as $\frac{2}{3} = \frac{6}{9}$.

One method for finding equivalent fractions is to use the Multiplicative Identity (Identity Property of Multiplication), that is, multiplying the given fraction by a form of the number 1 such as $\frac{2}{2}$, $\frac{5}{5}$, etc. In this course we call these fractions a “Giant One.” Multiplying by 1 does not change the value of a number.

For additional information, see the Math Notes box in Lesson 1.2.8 of the *Core Connections, Course 2* text.

Example 1

Find three equivalent fractions for $\frac{1}{2}$.

$$\frac{1}{2} \cdot \frac{2}{2} = \frac{2}{4}$$

$$\frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$$

$$\frac{1}{2} \cdot \frac{4}{4} = \frac{4}{8}$$

Example 2

Use the Giant One to find an equivalent fraction to $\frac{7}{12}$ using 96ths: $\frac{7}{12} \cdot \frac{?}{?} = \frac{?}{96}$

Which Giant One do you use?

Since $\frac{96}{12} = 8$, the Giant One is $\frac{8}{8}$: $\frac{7}{12} \cdot \frac{8}{8} = \frac{56}{96}$

Problems

Use the Giant One to find the specified equivalent fraction. Your answer should include the Giant One you use and the equivalent numerator.

1. $\frac{4}{3} \cdot \frac{?}{?} = \frac{?}{15}$

2. $\frac{5}{9} \cdot \frac{?}{?} = \frac{?}{36}$

3. $\frac{9}{2} \cdot \frac{?}{?} = \frac{?}{38}$

4. $\frac{3}{7} \cdot \frac{?}{?} = \frac{?}{28}$

5. $\frac{5}{3} \cdot \frac{?}{?} = \frac{?}{18}$

6. $\frac{6}{5} \cdot \frac{?}{?} = \frac{?}{15}$

Answers

1. $\frac{5}{5}, 20$ 2. $\frac{4}{4}, 20$ 3. $\frac{19}{19}, 171$ 4. $\frac{4}{4}, 12$ 5. $\frac{6}{6}, 30$ 6. $\frac{3}{3}, 18$

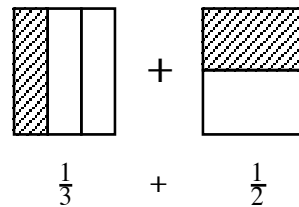
ADDITION AND SUBTRACTION OF FRACTIONS

Before fractions can be added or subtracted, the fractions must have the same denominator, that is, a common denominator. We will present two methods for adding or subtracting fractions.

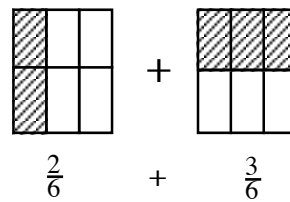
AREA MODEL METHOD

Step 1: Copy the problem. $\frac{1}{3} + \frac{1}{2}$

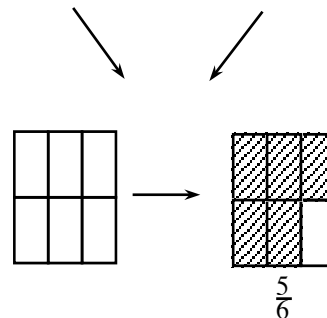
Step 2: Draw and divide equal-sized rectangles for each fraction. One rectangle is cut vertically into an equal number of pieces based on the first denominator (bottom). The other is cut horizontally, using the second denominator. The number of shaded pieces in each rectangle is based on the numerator (top). Label each rectangle, with the fraction it represents.



Step 3: Superimpose the lines from each rectangle onto the other rectangle, as if one rectangle is placed on top of the other one.



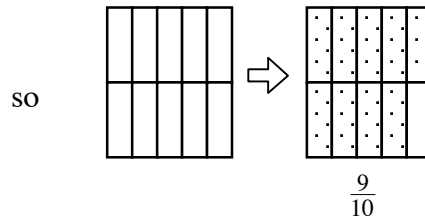
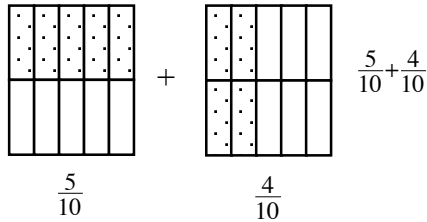
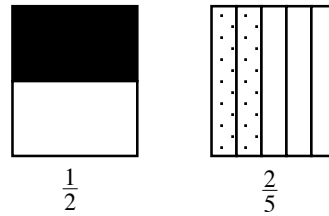
Step 4: Rename the fractions as sixths, because the new rectangles are divided into six equal parts. Change the numerators to match the number of sixths in each figure.



Step 5: Draw an empty rectangle with sixths, then combine all sixths by shading the same number of sixths in the new rectangle as the total that were shaded in both rectangles from the previous step.

Example 1

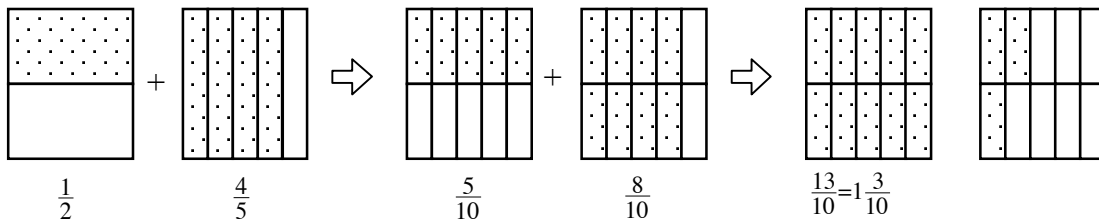
$\frac{1}{2} + \frac{2}{5}$ can be modeled as:



Thus, $\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$.

Example 2

$\frac{1}{2} + \frac{4}{5}$ would be:



Problems

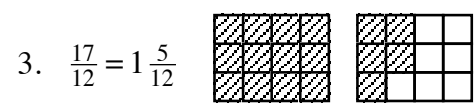
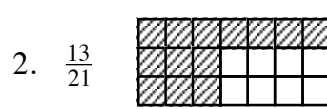
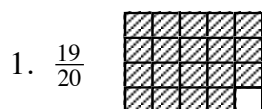
Use the area model method to add the following fractions.

1. $\frac{3}{4} + \frac{1}{5}$

2. $\frac{1}{3} + \frac{2}{7}$

3. $\frac{2}{3} + \frac{3}{4}$

Answers



COMPOUND PROBABILITY

Sometimes when you are finding a probability, you are interested in either of two outcomes taking place, but not both. For example, you may be interested in drawing a king or a queen from a deck of cards. At other times, you might be interested in one event followed by another event. For example, you might want to roll a one on a number cube and then roll a six. The probabilities of combinations of simple events are called **compound events**.

To find the probability of *either* one event *or* another event that has nothing in common with the first, you can find the probability of each event separately and then add their probabilities. Using the example above of drawing a king or a queen from a deck of cards:

$$P(\text{king}) = \frac{4}{52} \quad \text{and} \quad P(\text{queen}) = \frac{4}{52} \quad \text{so} \quad P(\text{king or queen}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

For two independent events, to find the probability of *both* one *and* the other event occurring, you can find the probability of each event separately and then multiply their probabilities. Using the example of rolling a one followed by a six on a number cube:

$$P(1) = \frac{1}{6} \quad \text{and} \quad P(6) = \frac{1}{6} \quad \text{so} \quad P(1 \text{ then } 6) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

Note that you would carry out the same computation if you wanted to know the probability of rolling a one on a green cube, and a six on a red cube, if you rolled both of them at the same time.

Example 1

A spinner is divided into five equal sections numbered 1, 2, 3, 4, and 5. What is the probability of spinning *either* a 2 *or* a 5?

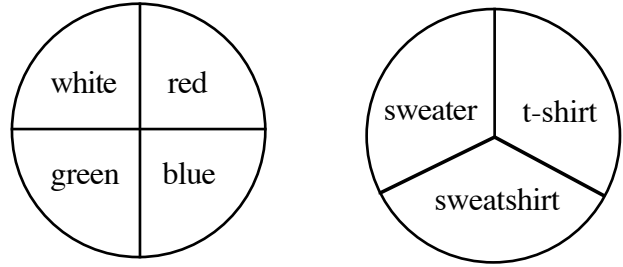
Step 1: Determine both probabilities: $P(2) = \frac{1}{5}$ and $P(5) = \frac{1}{5}$

Step 2: Since these are *either-or* compound events, add the fractions describing each probability: $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

The probability of spinning a 2 or a 5 is $\frac{2}{5}$: $P(2 \text{ or } 5) = \frac{2}{5}$

Example 2

If each of the regions in each spinner at right is the same size, what is the probability of spinning each spinner and getting a green t-shirt?



Step 1: Determine both possibilities:

$$P(\text{green}) = \frac{1}{4} \text{ and } P(\text{t-shirt}) = \frac{1}{3}$$

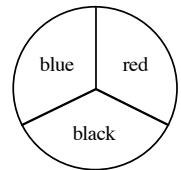
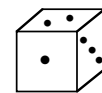
Step 2: Since you are interested in the compound event of *both* green *and* a t-shirt, multiply both probabilities: $\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

The probability of spinning a green t-shirt is $\frac{1}{12}$: $P(\text{green t-shirt}) = \frac{1}{12}$

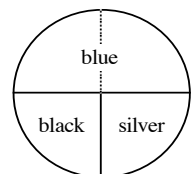
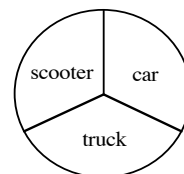
Problems

Assume in each of the problems below that events are independent of each other.

- One die, numbered 1, 2, 3, 4, 5, and 6, is rolled. What is the probability of rolling *either* a 1 *or* a 6?
- Mary is playing a game in which she rolls one die and spins a spinner. What is the probability she will get *both* the 3 *and* black she needs to win the game?



- A spinner is divided into eight equal sections. The sections are numbered 1, 2, 3, 4, 5, 6, 7, and 8. What is the probability of spinning a 2, 3, or a 4?
- Patty has a box of 12 colored pencils. There are 2 blue, 1 black, 1 gray, 3 red, 2 green, 1 orange, 1 purple, and 1 yellow in the box. Patty closes her eyes and chooses one pencil. She is hoping to choose a green or a red. What is the probability she will get her wish?
- Use the spinners at right to tell Paul what his chances are of getting the silver truck he wants.



6. On the way to school, the school bus must go through two traffic signals. The first light is green for 25 seconds out of each minute, and the second light is green for 35 seconds out of each minute. What is the probability that both lights will be green on the way to school?
7. There are 250 students at South Lake Middle School. 125 enjoy swimming, 50 enjoy skateboarding, and 75 enjoy playing softball. What is the probability a student enjoys all three sports?
8. John has a bag of jellybeans. There are 100 beans in the bag. $\frac{1}{4}$ of the beans are cherry, $\frac{1}{4}$ of the beans are orange, $\frac{1}{4}$ of the beans are licorice, and $\frac{1}{4}$ of the beans are lemon. What is the probability that John will chose one of his favorite flavors, orange, or cherry?
9. A nationwide survey showed that only 4% of children liked eating lima beans. What is the probability that any two children will both like lima beans?

Answers

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| 1. $\frac{2}{6}$ or $\frac{1}{3}$ | 2. $\frac{1}{18}$ | 3. $\frac{3}{8}$ |
| 4. $\frac{5}{12}$ | 5. $\frac{1}{12}$ | 6. $\frac{25}{60} \cdot \frac{35}{60} \approx 0.243$ |
| 7. $\frac{125}{250} \cdot \frac{50}{250} \cdot \frac{75}{250} = \frac{3}{100}$ | 8. $\frac{2}{4}$ or $\frac{1}{2}$ | 9. $\frac{1}{16}$ |